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INLET FOGGING OF GAS TURBINE ENGINES - PART B: FOG DROPLET SIZING ANALYSIS, NOZZLE TYPES, MEASUREMENT AND TESTING

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ABSTRACT

The inlet fogging of gas turbine engines for power augmentation has seen increasing application over the past decade yet not a single technical paper treating the physics and engineering of the fogging process, droplet size measurement, droplet kinetics, or the duct behavior of droplets, from a gas turbine perspective, is available. This paper provides the results of extensive experimental and theoretical studies conducted over several years, coupled with practical aspects learned in the implementation of nearly 500 inlet fogging systems on gas turbines ranging in power from 5 to 250 MW. Part B of the paper treats the practical aspects of fog nozzle droplet sizing, measurement and testing presenting the information from a gas turbine fogging perspective. This paper describes the different measurement techniques available, covers design aspects of nozzles, provides experimental data on different nozzles and provides recommendations for a standardized nozzle testing method for gas turbine inlet air fogging.

NOMENCLATURE

A_d	Surface Area of the Droplet (m^2)
AD or D21	Absorption Diameter (m)
AMD or D10	Arithmetic Mean diameter (m)
ASTM	American Society for Testing and Materials
CV	Concentration Volume (ppm)
D_d	Droplet Diameter (m)
ED or D31	Evaporative Diameter (m)
Fpm	Feet per Minute
MMD or Dv50	Mass Median Diameter (m)
PDPA	Phase Doppler Particle Analyzer
PMS	Particle Measurement system
RSF	Relative Span Factor
SAMD	Surface Area Mean Diameter (m)
SMD or D32	Sauter Mean Diameter (m)
V_d	Droplet Volume (m^3)
VMD or D30	Volume Mean Diameter (m)
V_{rel}	Droplet relative velocity ($m.s^{-1}$)
We	Weber Number
γ_w	Surface tension of the water ($N.m^{-1}$)
ρ_a	Density of the air ($kg.m^{-3}$)

INTRODUCTION

Over the past decade and especially over the past five years, the application of inlet fogging for the power augmentation of gas turbines has become increasingly popular. It is estimated that approximately 700 gas turbines worldwide have fogging systems at this time including several new F-class gas turbines. To this point however, not a single technical paper exists comprehensively covering the physics and engineering of the fogging process, droplet measurement, kinetics, duct behavior of droplets from a gas turbine perspective.

This paper provides the results of extensive experimental and theoretical studies conducted over several years, and also mentions practical aspects learned in the implementation of nearly 500 inlet fogging systems on gas turbines. A major problem faced by gas turbine users considering the utilization of inlet fogging is that different fog nozzle manufacturers and suppliers present data in very different formats and under different operating conditions. In this paper we point out the key operating parameters that are pivotal in a gas turbine fogging application and also provide data on a large number of nozzle tests made in our experimental wind tunnel.

MEASUREMENT APPROACHES TO DETERMINE FOG DROPLET SIZE

The importance of the determination of accurate droplet size for gas turbine inlet fogging applications was covered in Part A of this paper. The atomization process of any type of fog nozzle consists of the conversion of the kinetic energy of water under pressure into a spray that contains a wide range of statistically distributed droplet sizes. Different techniques exist to measure particles size. For gas turbine inlet fogging applications, imaging (microscopy and the use of high speed video cameras) and the laser light scattering techniques are commonly used.

There are two techniques that use light scattering principles: the spatial technique and the flux technique.

The spatial technique instantly samples a large number of droplets in a given volume. This is a number-density-weighted technique (Malvern and Imaging Systems are two well known manufacturers of this type of equipment).

The flux or temporal technique samples and counts individual droplets passing through the sampling volume (established by the cross section of two beams of laser light) in a given time interval and is a number-flux-weighted technique. PDPA and PMS are two systems that use this technique.

With the flux measurement technique, ignoring a droplet of 100 microns, is the same as ignoring 1000 droplets of 10 micron (since volume is a cube function of the diameter). In gas turbine fogging applications therefore, ignoring a relatively significant number of small droplets does not significantly affect the distribution; however the distribution is drastically altered if one ignores droplets of a larger diameter. In the measurement of fog droplet size, an important source of error derives from the conversion: if the error in the measurement of the mean diameter is 5%, an error of 125% will be propagated when this result is converted to the volume, since the volume or mass mean diameter is a cubic function of the mean diameter. This problem occurs when converting from a measured diameter to a calculated volume as is done with a flux type system.

Due to the difference in the droplet's velocity at the sampling region, the two techniques may produce different results as stated by Dodge [1]. In examining measurement approaches for gas turbine fogging applications, it is clear that a spatial sizing technique (working with volumes) and analyzing a huge number of droplets instantly is more appropriate as we are dealing with billions of droplets.

Teske et al [2] compared droplet sizes measured using five different measurement techniques: Malvern, Particle Measurement System, Phase Doppler Particle Analyzer, Hot Wire, and Video Imaging and observed a similarity between the measured data from Malvern, PDPA and Imaging Video Systems. He found that, the agreement between several Malvern instruments was superior to the agreement between several PDPA instruments.

Similar studies conducted by the British Crop Protection Council [3] and others [4,5,6,7,8] confirmed the preciseness, repeatability and measurement stability of the Malvern system compared to other measurement systems when measuring the droplet size distribution of spray atomized using different types of nozzles.

Based on our extensive experience in the water atomization and droplet measurement fields, and on results confirmed by many researchers, we have standardized on the use of a Malvern Spraytec system to characterize gas turbine inlet air fogging nozzles.

The Malvern system is a non-intrusive system. It instantly samples a large number of droplets occupying a given volume. This laser diffraction system, along with an algorithm allowing us to correct for multiple scattering errors, was used to measure the size distribution of the droplets throughout this paper.

A photograph of the experimental setup using a Malvern Spraytec system is shown in Figure 1.

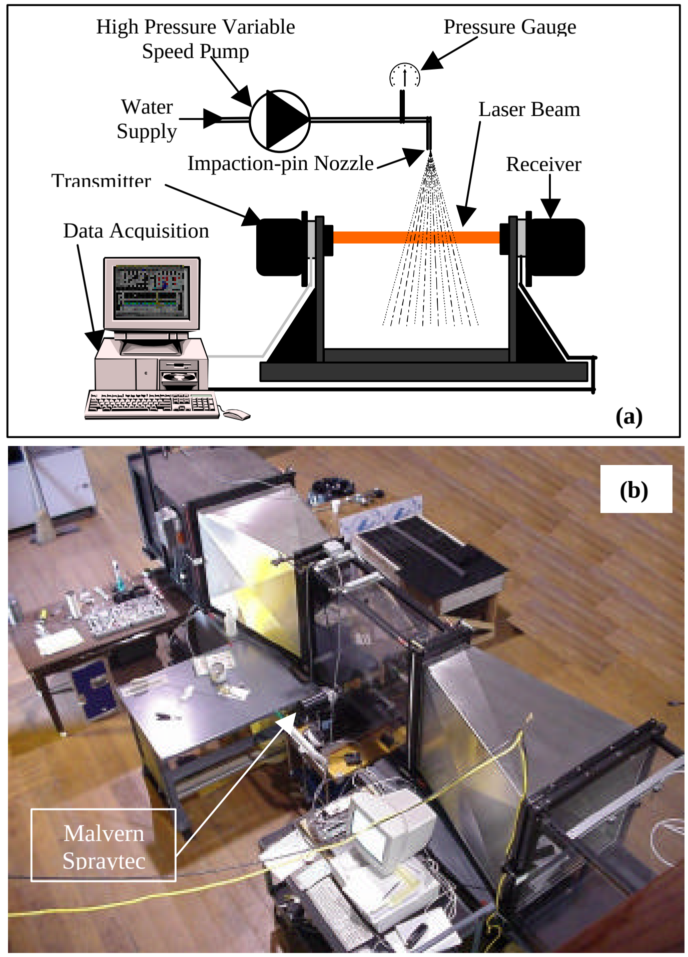

Figure 1. (a) Experimental Setup, in still air and (b) wind tunnel section showing Malvern Spraytec Measurement System for nozzle testing under airflow conditions

DROPLET DIAMETER DEFINITIONS

It is important for anyone evaluating the relative performance of inlet air fogging nozzles to understand the terminology used to characterize nozzle performance. All atomizing nozzles produce a range of droplet sizes so, in order to characterize a nozzle with a single value, it is necessary to take some statistical function of the droplets measured. Such statistical functions produce a value that refers to a droplet diameter, which in some way describes or characterizes the total spray. The stated diameter is possibly fictitious because none of the droplets measured may actually have the stated diameter.

In reality, inlet air fogging nozzles produce several billion droplets per second. Therefore, in order to ensure reasonable results, it is important that any measurement technique measure a large number of droplets. Furthermore, all nozzles produce, to a limit, smaller droplets as the pressure is increased. Therefore, to be of practical use any statement of the size of droplets produced should be accompanied by the operating pressure of the nozzle. As was described in Part A of this paper, one has to consider the definition of the various types of droplet diameters as well as the *conditions* under which measurements were taken. Different measurement conditions may produce very different droplet size results. The effect of measurement conditions such as airflow velocity and location of the measurement in the spray plume will be discussed in detail ahead.

Some common diameters (Mean Diameters and Representative Diameters) used are presented below.

MEAN DIAMETERS

Detailed formulae for the Mean Diameters definitions are provided in Appendix A.

Arithmetic Mean Diameter (AMD) D_{10}: The simple average diameter of all the droplets in a spray. D10 is equal to the sum of the diameter of all the droplets divided by the quantity of droplets.

Surface Area Mean Diameter (SAMD) D_{20}: The droplet diameter is measured directly using an imaging system. The SAMD diameter is then calculated on the basis of the droplets surface area. By taking the square of the diameter of each droplet, it is possible to calculate its surface area:

$$A_d = \pi \times D_d^2 \tag{1}$$

An average surface area is then calculated using the sum of the surface areas of all the droplets divided by the number of droplets. The square root of the average surface area is then divided by π to get the SAMD. The SAMD value characterizes the spray by giving the diameter of a hypothetical droplet that has a surface area equal to the average surface area of all the measured droplets.

Volume Mean Diameter (VMD) D_{30}: This characteristic diameter is calculated on the basis of the droplet's volume. The volume of a droplet is given by:

$$V_d = \frac{\pi \times D_d^3}{6} \tag{2}$$

Using this equation, it is possible to calculate the volume of each droplet, then the average volume by taking the sum of the volume of all the droplets and dividing it by their number, and then to finally take the cubic root of the averaged volume using equation (2) to get the VMD. The VMD value characterizes the spray by giving the diameter of a hypothetical droplet that has a volume equal to the average volume of all the measured droplets.

Sauter Mean Diameter (SMD) D_{32}: This diameter is calculated using the concept of the volume to surface area ratio. It is equal to the sum of the cube of all diameters divided by the sum of the square of all diameters. This yields a characteristic droplet diameter that has a volume-to-surface-area ratio equal to the volume-to-surface-area ratio of the entire spray. This diameter is particularly important in gas turbine evaporative fogging system applications because the mass transfer happens at the interface of the droplets and the surrounding air (i.e. at the droplet surface). To enhance the evaporation of a population of droplets, one has to maximize the active surface areas and minimize the internal volumes.

Absorption Diameter (AD) D_{21}: This diameter is calculated using the surface to diameter ratio concept. It is equal to the square root of the sum of the square of all the droplets diameters divided their straight sum.

Evaporative Diameter (ED) D_{31}: This diameter is calculated using the volume to diameter concept. It is equal to the sum of the cube of all the droplets diameters divided by their straight sum.

REPRESENTATIVE DIAMETERS

There are other representative diameters, which can be easily measured from cumulative distribution[1] curves. These are defined as:

***Dv01* (also known as Dv10):** Is a representative diameter where 10% of the total volume of the liquid sprayed is made up of droplets with diameters smaller or equal to the stated value.

***Dv05* (also known as Dv50):** this is the same as the Volume Median Diameter or Mass Median Diameter (MMD), assuming water. This is the representative diameter where 50% of the total volume of the liquid sprayed is made up of droplets with diameters larger than the stated value and 50% is made up of droplets with diameters smaller than the stated value.

***Dv09* (also known as Dv90):** this is the representative diameter where 90% of the total volume of the liquid sprayed is made up of droplets with diameters smaller than or equal to the stated value. This representative diameter is commonly used to characterize gas turbine inlet air fogging nozzles because there is concern that the ingestion of large droplets by the axial flow compressor might cause blade erosion or blade coating wear.

Other median diameters may be used, depending on the application, such as the number median diameter D_{N50} and the surface median diameter D_{S50}.

***RSF* Relative Span Factor:** this is a dimensionless parameter indicative of the uniformity of the drop size distribution. It is given by:

$$RSF = \frac{D_{v09} - D_{v01}}{D_{v05}} \tag{3}$$

FOG SPRAY HISTOGRAMS

Figures 2 shows two histograms (a and b) which are output from the Malvern laser particle analyzer software. The vertical, blue-colored bars represent "bins" of droplet sizes, which can be read off the abscissa. The volume frequency (as a percent of the total volume) can be read off the axis on the right side. Examining Figure 2a, we see that the droplets in the 10-micron bin represent about 4% of the total volume of the spray.

The red colored, s-shaped curve shows the cumulative volume across the size bins. Examining 2a again, and reading from the cumulative volume axis (on the left) we see that 50% of the volume of the droplets are about 18 microns or smaller (i.e. the Dv50, or volume/mass mean diameter is 18 microns).

The charts also give:

- Transmission, which is the percent light transmission through the spray. If this value is below a certain limit, the spray density may be too high and multiple-scattering errors may result.
- CV, which is the concentration volume given in parts per million. CV also gives an indication of liquid water content (1 CV is equal to one cubic centimeter of liquid water per cubic meter of air). CV plays an important role in our proposed standard characterization method for gas turbine

[1] Droplet size distribution in the spray is characterized by Rosin-Rammler expression and function of two parameters, which are the measure of the range of droplet sizes and a representative diameter.

inlet air fogging systems, given in the last section of this paper.

- Specific Surface Area, which is the surface area of the spray, expressed as square meters per cubic centimeters of water.
- Dv10, Dv50, Dv90 and D_{32} (SMD), all of which are defined above.
- D_{43}, which is the Mean Mass or De Brouckere's or Harden's diameter used in combustion equilibrium applications.
- Span, which is the relative span factor as defined above.

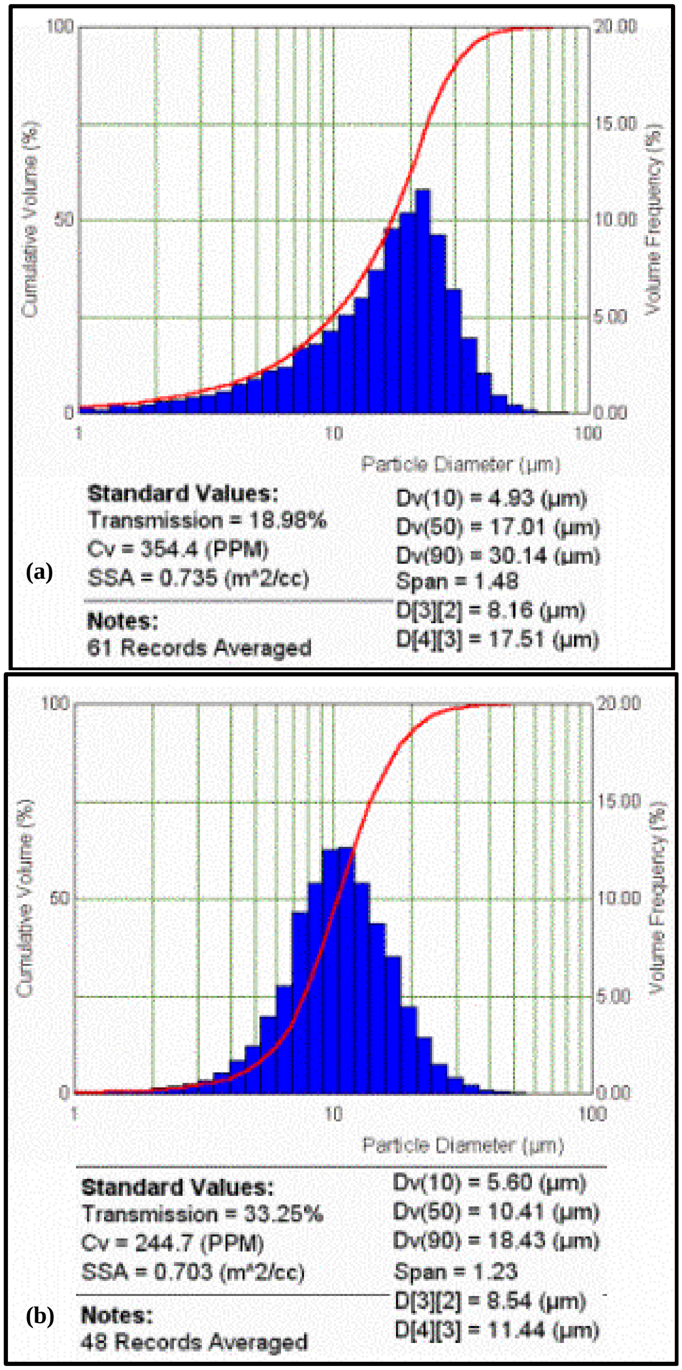

Figure 2. Two droplet measurement histograms showing the importance of the use of both SMD and Dv90. Data is from actual experiments done on nozzles

APPROPRIATE DIAMETER NUMBERS FOR CHARACTERIZING AND COMPARING INLET AIR FOGGING NOZZLES

The most important quantitative factors for inlet air fogging are the surface area of water exposed for evaporation (affects the evaporative cooling efficiency of the spray) and the size of the largest droplets (affects the potential for compressor blade distress and the amount of water that falls out in the duct). Given the above, the SMD and Dv90 numbers are of primary significance for gas turbine inlet-air cooling.

Evaporation rate is a strong function of the surface area of water exposed to the air. SMD expresses the total surface area of water exposed for a given volume of water sprayed. Looking again at Figure 2, we see that there is about a 5% difference in SMD (given on the chart as D[3][2]) and, as would be expected, there is also about a 5% difference in specific surface area—square meters of surface per cubic cm of water atomized (SSA on the chart).

The importance of knowing both SMD and Dv90 can be seen in Figure 2. As noted above, the SMD in both histograms is approximately the same (about 8 microns). However, in the first histogram (2a), there are a larger number of big droplets (Dv90 about 30 microns) while in the second histogram (2b) Dv90 is about 18 microns—nearly half that of the first. Thus, in order to meaningfully evaluate nozzles, one *has* to know the Dv90 number as well as the SMD.

In conclusion we feel that for gas turbine fogging applications, the SMD and Dv90 numbers are of significance.

Further to this, the *location* of the measurement in the spray plume, and other measurement conditions such as airflow velocity, are also very important. Measurements taken in different locations can yield very different results. In the sections ahead we provide a recommended standard for the measurement of droplets.

It is interesting to note that measurements of droplet diameters at distances progressively further downstream from the fogging nozzles (in an air stream that is under-saturated) results in progressively larger diameters. This might lead one to assume that droplet coalescence was occurring. In fact, what is occurring is that the smallest droplets evaporate resulting in the average diameter of the remaining population increasing.

Consider for instance a spray consisting of just one 10-micron water droplet and one 50-micron droplet. The larger droplet has about 125 times the mass and about 25 times the surface area of the 10-micron droplet. Now imagine removing, by evaporation, the mass of one 10-micron droplet from both droplets. The smaller droplet disappears but the diameter of the larger droplet reduces from 50 microns to just 49.9 microns.

Evaporation rate is, of course, a strong function of surface area and throughout the evaporation process, the 50 micron droplet will always have more surface area than the ten micron droplet as it started with 25 times more surface area. Even if 25 times more mass is removed from the larger droplet, it would still be more than 46 microns in diameter. With the removal of 100 times more mass the larger droplet would still be almost 30 microns in diameter.

To visualize the impact of this effect on a large population of droplets, consider a spray of billions of droplets, half at 10-microns and half at 50-microns. The average droplet is 30-microns. Now evaporate away the 10-micon droplets, and reduce the mass of the larger droplets, by 25 times the mass of a 10-micron droplet, and we are left with a spray having billions of droplets larger than 46-microns. The average droplet size of the spray increased from 30 to 46-microns yet no coalescence occurred.

This phenomenon was proved in the wind tunnel, described in Part C of this paper, by showing that agglomeration was not occurring. The airflow in the wind tunnel was saturated, so that evaporation would not occur, and droplet size was measured at various points downstream of the nozzles. The phenomenon can also be shown using the empirically validated evaporation rate model, also given in Part A of this paper.

As mentioned above, the *location* of the measurement in the spray plume and other measurement conditions such as airflow velocity, are also very important. Figure 2 demonstrates this fact, because the two histograms are in fact the same nozzle operating at the same pressure but represent measurements take at different airflow velocities and at different locations in the spray plume. In the sections ahead we provide a recommended standard for the measurement of droplets for gas turbine inlet air fogging systems. The standard makes it possible to make a meaningful comparison of different fogging nozzles.

A Suggested Representative Diameter. The importance of considering both SMD and Dv90 when comparing different fogging nozzles suggests the possibility of adopting a single representative diameter that would take both factors into account and would thus be more appropriate for quantifying gas turbine inlet air fogging sprays. One very simple approach would be to take an average of the SMD and Dv90 values for a given spray. For the sprays depicted in histograms of Figure 2a and 2b this simplistic approach yields diameters of 19.15 microns and 13.49 microns, respectively. The result is a single representative diameter that gives a good indication as to which nozzle is most suitable for gas turbine inlet air fogging. We are working on statistical approaches to refine and validate this parameter and a more detailed study of a suitable representative diameter will be given in a later paper.

NOZZLE DESIGN

For gas turbine inlet air fogging applications, the goal of atomization is to disintegrate the water into the smallest droplet sizes possible in order to maximize the surface area so as to increase the heat transfer between the droplet surface and the air. The underlying principle of any pressure-atomizing nozzle is to convert the applied pressure into kinetic energy by the use of a small orifice.

When selecting fog nozzles for inlet-air cooling some engineering compromises have to be made. There may be a cost and maintenance benefit associated with using fewer nozzles with higher flow rates but, on the downside, higher flow rate nozzles have to be operated at higher pressures (additional cost) or they make larger droplets (less effective cooling, potential for blade stress) and fewer nozzles means fewer points of fog emission (potential for uneven cooling at the compressor face and less effective cooling of the entire air mass flow).

The most important factors for nozzle selection are droplet size and the distribution of fog in the duct cross-section, as these affect both the efficiency of the fog cooling process and the safe operation of the turbine.

If the nozzle pattern at the manifolds allows a significant portion of air to pass by without mixing with the fog spray plumes, the result will be less evaporation before the air reaches the compressor inlet.

There are two types of nozzles which are in general use for gas turbine fogging applications: impaction-pin nozzles and swirl-jet nozzles, although impaction-pin designs predominate. A photograph of the two types of nozzles is shown in Figure 3. It is very important to note that the performance of any nozzle is a function of its specific design and therefore any comparative testing must be done with the specific design contemplated for use. A general discussion of the two design philosophies is presented in this section. Later, comparative tests will be shown with specific nozzles that have been used for inlet air fogging applications.

Figure 3. Swirl and Impaction-pin Nozzles

Impaction-Pin Nozzles

With impaction-pin nozzles, water is forced through a smooth orifice and the high-velocity jet is directed at an impaction-pin located above the orifice. The velocity of the jet, and the size of the droplets produced, depends on the applied pressure. The impaction of the water jet on the pin results is a thin sheet of water in a conical shape. As the conical sheet extends away from the nozzle orifice the surface area of the sheet expands so that the water-sheet becomes increasingly thinner. Eventually the surface tension forces eventually cause the sheet to break down into "fingers" of water and aerodynamic instabilities cause the fingers to break into ligaments and then into droplets. The thickness of the sheet of water depends on the orifice hole diameter, the impaction-pin geometry and the operating pressure. Increasing the pressure results in an increase in velocity causing the sheet to extend and the thickness to decreases resulting in a decrease in droplet size as well as an increase in the flow rate of the nozzle[2].

A nozzle with a smooth and shallow orifice will have low frictional losses and will, therefore, be more efficient at converting the energy of the applied pressure to axial velocity and hence will make a thinner sheet and smaller droplets. It is important that nozzles be manufactured with minimal variations as a small difference in orifice diameter, orifice smoothness or depth, will result in different flow rates and different droplet sizes. It is therefore important to measure the droplet size of many nozzles from any given manufacturer and to examine the statistical spread.

The velocity of the water jet produced by any direct-pressure nozzle is a function of the orifice size geometry and the fluid pressure. Jet velocity is also a function of the viscosity and density of the pumped fluid and the point at which the sheet disintegrates into ligaments and then to droplet (and therefore the size of the droplet produce) is a function of both jet velocity and surface tension of the fluid. However, for purposes of comparing inlet-fogging nozzles, the authors have largely ignored density, surface tension and viscosity effects, since the pumped fluid is always demineralized water essentially at ambient temperature.

Due to the high velocity of the droplets very close to the nozzle tip, the Weber number is high and this leads to an important secondary break up of the larger droplets. The result of the collision between droplets in this area is a shattering due to the high Weber number. Details of high Weber number droplet-shattering effect are provided ahead.

[2] There are practical design optimization issues involved with increasing the operating pressure. These issues include, the thickness of the orifice plate that would then increase frictional losses, and also droplet coalescence agglomeration effects (increasing pressure also increases the density of droplets near the orifice and, hence, increases the probability of collision and coalescence). There are also the practical limitations of pumping and piping issues when very high pressures are considered.

It is helpful to consider three different locations (zones) in the fog plume when considering the question of the probability of droplet collision and the resulting separation, coalescence and bouncing:

1. The reflexive and stretching separation Zone: Very near the nozzle orifice droplet velocities are very high (resulting in Weber numbers as high as 50) and droplet density is very high. Collision of droplets is highly probable but mostly results in stretching and reflexive separation.
2. The Coalescence and bouncing Zone: Moving further from the orifice, the droplets move a bit slower (lower Weber number) but droplet density is still high. Collision of droplets is somewhat less probable but still occurs and the result of collision is coalescence or agglomeration.
3. The non-Collision Zone: Moving still farther from the orifice, the droplets move much slower and the Weber number is very low. As the plume has spread out, the droplet density is low and the likelihood of collision is almost nil. Further, droplet collision at low relative velocity would result in bouncing, and not coalescence.

The relative size of these zones depends on the specific nozzle design, the fluid pressure and the velocity of the air that the nozzle is spraying into.

Nozzle designs should be optimized for the specific application. Figure 4 shows visual plume shape of different makes of impaction-pin nozzles. The nozzle in the center shows a wider plume (better coverage in the duct cross-section) and a more homogeneous distribution.

Figure 4. Visual plume shape of different makes of impaction-pin nozzles at an operating pressure of 138 barg (2000 psig). Mee nozzle (152 micron orifice, 316 stainless steel construction) in the center

The orifice diameter and operating differential pressure of an impaction-pin nozzle governs the flow rate. Impaction-pin nozzles with different orifice diameters and their resultant plumes are shown in Figure 5. Diameters are 5.8-mils (147-microns) on the left through 18-mils (457-microns) on the right. Optimal design, from a flow rate, droplet size, and plume dimension standpoint for gas turbine applications, was found to be 6-mil (152-micron) diameter (2nd nozzle from the left).

Figure 5. Various impaction-pin nozzles, from 147-micron (on right) to 457-micron orifice diameter (left), operating pressure 138 barg

Impaction-pin nozzles exhibit what is often called the “sprinkle” effect. By this we mean that some droplets tend to hit the pin and form larger droplets. However, these are either broken up (due to the high Weber number) or fall out immediately and, in any case, they represent a very small portion of the total volume of the spray.

The distribution of the droplets across the cross section of the plume is complicated. Due to the presence of the impaction-pin, the shape of the plume is not symmetrical as can be seen in Figure 6 (view from the impaction-pin side). The view from the other side is shown in Figure 7. As can be seen in these figures, the shading effect of the impaction-pin is very local to the nozzle tips and therefore has little effect on the overall fog plume.

Figure 6. Impaction-pin nozzle showing small shading effect from impaction-pin

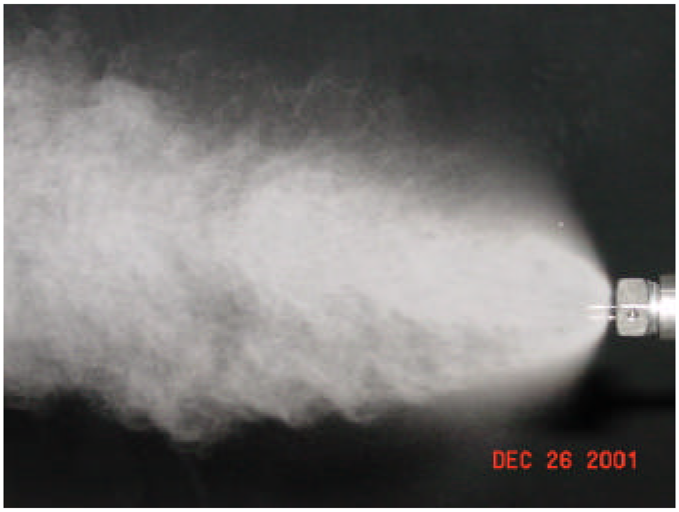

Figure 7. Opposite view of the nozzle shown in Figure 6

Secondary Breakup -Weber Number Effect The secondary break-up of liquid droplets occurs if the surface tension force, which acts to keep the droplet spherical, is no longer able to balance the opposing aerodynamic forces due to the high relative velocity between the droplets and the surrounding airflow. The ratio of the aerodynamic and surface tension forces is the dimensionless Weber number given by this equation:

$$We = \frac{r_a \cdot v_{rel}^2 \cdot D_d}{g_w} \quad (4)$$

For water, with a critical Weber number value of 6 taken over large interval of Ohnesorge number [9], secondary break-up of water droplets may occur. Details regarding secondary breakup may be found in Pilch and Erdman [9]. This reference provides the use of breakup time data and velocity history data to predict the maximum size of stable fragments for acceleration-induced breakup of liquid droplet. It is important to note that the Weber number effect applies to both impaction-pin and swirl nozzles but has a greater significance with impaction-pin nozzles due to the higher relative velocities of droplets under similar operating conditions (i.e. fluid pressure and air velocity). The Weber number also has other applications for understanding the shattering of droplets in intake ducts as will be discussed in Part C of this paper.

Swirl-Jet nozzles

With swirl-jet nozzles, the fluid is forced to enter tangentially into the swirl chamber taking a helical path before discharging through a cylindrical hole concentric to the swirl chamber. This leads to the formation of an air core with a larger diameter in the swirl chamber, as compared to the one in the orifice. The swirling jet results in the formation of an axisymmetrical, thin conical water sheet. The conical sheet breaks down in to ligaments and then small droplets. Because of the swirling nature of the flow, there are both axial and radial forces involved. Unlike the impaction-pin nozzle the ligaments are formed perpendicular to the axis of the nozzle (due to the swirling action of the conical sheet). As the operating pressure is increased, both the radial and axial forces increase. This phenomenon occurs up to a point after which an increase of the operating pressure causes the axial velocity to dominate and the radial forces drop causing the nozzle to operate as an "open orifice" nozzle, i.e. with increasing pressures the cone angle decreases and eventually disappears. The maximum cone angle depends on the geometrical characteristics[3] of the swirl nozzle. This result is in accord with the findings of Dupouy et al [10]. It is also important to note that the swirl action creates tangential forces, which induces frictional losses. This results in a reduced flow-efficiency, so that a swirl-jet nozzle with the same flow rate as an impaction-pin nozzle must have a larger orifice and will make larger droplets.

With a swirl-jet nozzle, continuously increasing the operating pressure results in a decrease in droplet size only to the point that the swirl phenomenon breaks down (i.e. the nozzle begins to act like an open orifice). An impaction-pin nozzle, however, makes smaller droplets as the pressure is increased.

DROPLET SIZE MEASUREMENT

Location of Measurement

A large variation in droplet size, and size distribution, is found depending on the location in the plume where the measurements are taken. The droplet size at the center of the plume is smaller than the size at the edge. As the droplets leave the tip of the nozzle at high velocity, the induced airflow creates a draft toward the center of the plume. This draft carries the smallest droplets to the center of the plume, since smaller droplets follow the velocity and the direction of the airflow—droplets under 10 microns immediately follow the trajectory of the airflow. Consequently, measuring the size at the edge of the plume and at the center, gives a completely different droplet size and size distribution.

Figure 8 shows the distribution of the droplet size inside the plume in still air and in an airflow velocity field for an impaction-pin and a swirl-jet nozzle[4]. Figure 8 is for an impaction-pin nozzle in still air (Figures 8a & 8b) and under airflow velocity (Figure 8c & 8d). Results for a swirl-jet nozzle are shown for still air in Figure 8e and 8f. These plots show both the Dv90 diameters (on the right) and the SMD (D32) diameters (on the left). The plot uses a color continuum to represent droplet sizes. In this plot, the smallest droplet sizes are represented by blue and the largest by red. The different color dots[5] show the size heterogeneity in the distribution of the droplets, in the R (radial from the orifice) and Z (axial from the orifice) directions.

The charts in Figure 8 clearly show that:

- Droplet size is a strong function of the measurement location.
- Smallest droplet sizes are found in the center of the plume near the exit of the nozzle.
- The smaller drops evaporate quickly, leaving only the larger droplets as axial distance increases.
- In the radial direction, droplet size increases as we move to the edge of the plume because the small droplets quickly follow the airflow and move to the center while the larger droplets maintain their initial trajectories and stay in the edge of the plume.

Size is affected by air velocity as can be seen by comparing Figure 8a and 8c. For example, the maximum diameter decreased from 21.35 microns to 16.54 microns. This is due to the fact that airflow separates the small droplets and large droplets into different trajectories, so the larger droplets, which initially have a higher velocity, do not collide and coalesce with the smaller droplets. All

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[3] Ratio of swirl-chamber diameter to final orifice diameter and L/D ratio of the swirl chamber, and the L/D ratio of the final orifice.

[4] Swirl-jet Nozzle: 152-micron (0.06 inch), flow rate of 7.5 l/h (0.033 gpm) at 138 barg (2000 psig). Impaction Pin Nozzle: 152-micron (0.006 inch) orifice diameter, flow rate of 10.2 l/h (0.045 gpm) at 138 barg (2000 psig).

[5] The physical sizes of the dots on this plot are also proportional to the diameters.

droplets take the same velocity within several centimeters of the orifice, and the density of droplets is much less as the plume expands, so that there is little probability of coalescence further from the orifice.

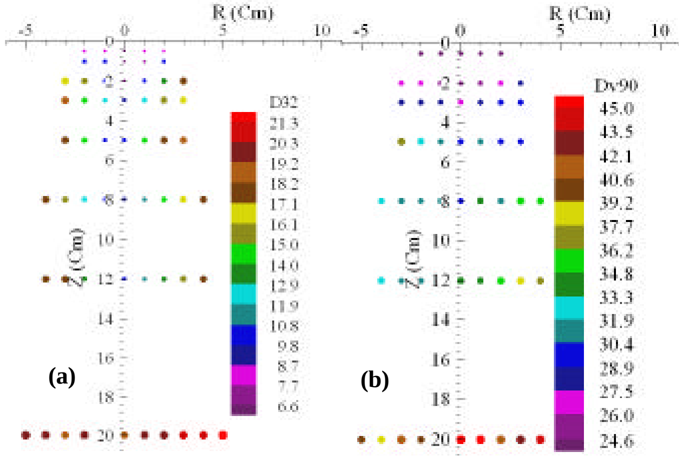

Figure 8-a and 8-b. Impaction-pin droplet size experimental results for still air, operating pressure 2000 psig (138 barg)

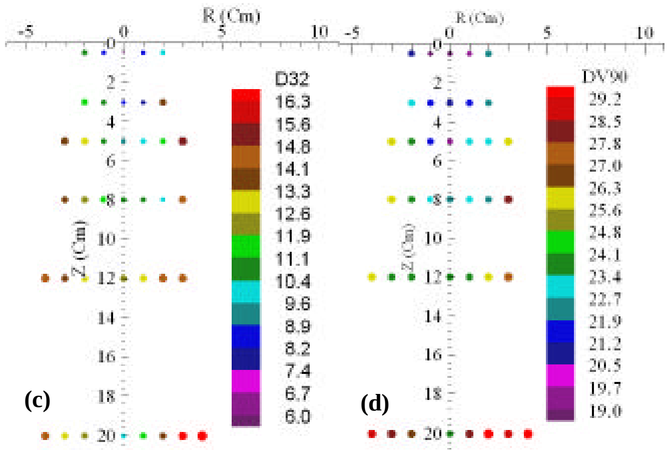

Figure 8-c and 8-d. Impaction-pin droplet size experimental results for airflow velocity of 900 fpm (4.6 m/sec), operating pressure 2000 psig (138 barg)

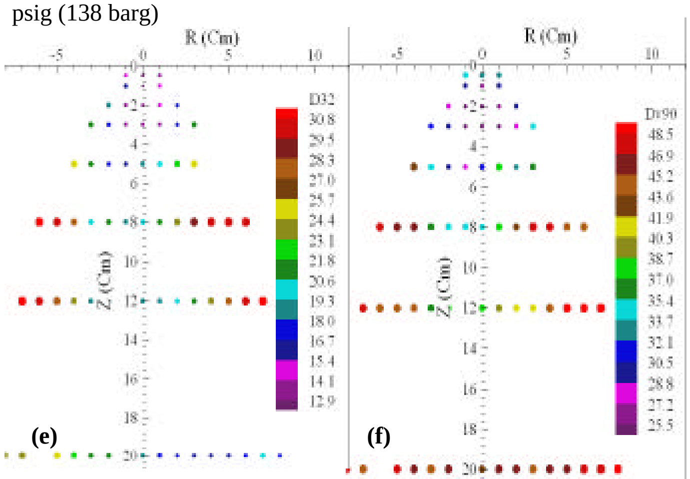

Figure 8-e and 8-f. Swirl-jet Nozzle droplet size experimental results for still air, operating pressure 2000 psig (138 barg)

Comparing the droplet size between impaction-pin and swirl-jet nozzles (Figure 8a & 8b with 8e & 8f) tested in still air, we can see that both D32 and Dv90 sizes are much smaller in the case of impaction-pin nozzle, even though the impaction-pin nozzle has a flow rate approximately 25% higher than the swirl-jet nozzle.

EFFECT OF VELOCITY ON DROPLET SIZE

In order to explain the effect of velocity on droplet size, experimental tests were conducted in our wind tunnel with impaction-pin nozzles operating at 138 barg. The spatial location definitions of the measurement points Z and R are shown in Figure 9. Figure 10 shows the results of tests when wind tunnel velocities are varied between 0 and 2500 ft/min (0-12.7 m/sec).

Readings were taken at different radial and axial positions in the fog plume. Each graph in Figure 10 has an indicator such as “R-1Z0.5”. This means that the measurement point is taken at a location of 1 cm above the plume centerline and at an axial distance of 0.5 cm from the nozzle. In Figure 10, Z has been fixed at 0.5 and R is at –1, 0 and +1 cm respectively for Figure 10 a, b and c. Thus Figure 10 a, b, and c represent measurements near the nozzle at different radial locations. The set of Figures 10 d, e, and f are at an axial distance of 12 cm (Z12) and radial locations of –3, 0, and +3 cm. Thus Figure 10 d, e and f represent measurements at some distance from the nozzle. In each of the figures, the abscissa is the droplet diameter in microns. The axis to the left is the cumulative volume in % (the S-shaped curves relate to this) while the axis on the right is the volume Frequency (the bell shaped curves relate to this). The different color lines all relate to measurements taken at velocities of 0 fpm, 1000 fpm and 2500 fpm (0 m/sec, 5.1m/sec and 12.7 m/sec).

In order to understand how these diagrams can be used, we provide the following examples. In figure 10a, if we enter the chart at a cumulative volume of 90 (on the left side), and look at the red S-shaped curve (velocity of 0 fpm) we can read from the abscissa a diameter of 20 microns. This means that 90% of the spray volume is contained in droplets with diameters less than 20 microns or, in other words, the Dv90 diameter is 20 microns. Similarly the Dv50 (mass median diameter) can be read to be approximately 15 microns.

In Figure 10e, if we enter the chart at 30 microns on the abscissa, and go to the volume frequency bell shaped curve for 0 fpm, we see that approximately 10% of the volume of the spray (or of the mass, since we are in all cases dealing with demineralized water) is droplets with a diameter of about 30 microns [6].

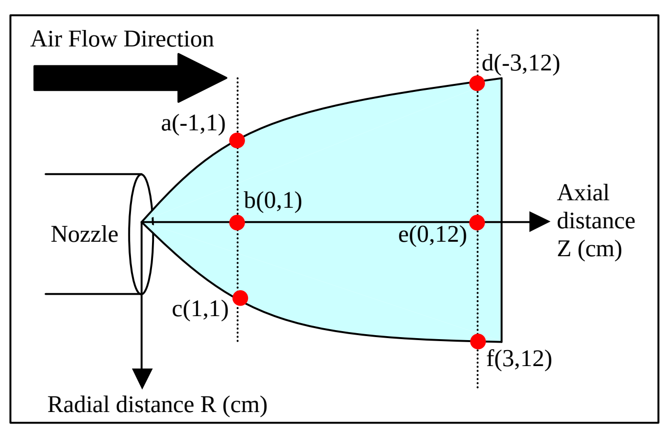

Figure 9. Spatial Location for Experimental Testing

[6] The droplets are in a bin or class of 29-30 microns.

Table 1, shows the Sauter Mean Diameters for the different velocities and for the different locations tested in Figure 10.

Velocity / Position	0 fpm 0 m/s	1000 fpm 5.1 m/s	2500 fpm 12.7 m/s
R-1Z0.5	10.29	9.4	7.94
R0Z0.5	8.45	6.96	5.99
R1Z0.5	10.82	9.19	9.24
R-3Z12	15.38	11.77	9.7
R0Z12	15.18	12.58	10.88
R3Z12	16.36	14.41	12.83

Table 1. Sauter Mean Diameters (microns) for the different locations and at different flow velocities for the nozzle tested in Figure 10

Observations on the Effect of Location and Velocity on Droplet Size

Based on Figure 10, the following important observations can be made:

- The cumulative volume (the s shaped curves) exhibit very small variations for changes in air flow velocity when measurements are made near the nozzle as can be seen in Figure 10a.
- This difference increases as the axial distance Z increases as can be seen in Figure 10d, e and f.
- When the Z value is high, i.e., when measurements are made far from the nozzle in the axial direction, the bell shaped curves tend to move to the right, i.e. the smaller droplets have evaporated and only larger diameters remain.
- In examining Figure 10d, it can be seen that with increasing airflow velocities the droplet size distribution move to the left (decrease) and so the cumulative volume (S curves) shifts to the left as expected.
- As the velocities increase, the droplet diameters drop. For example in Figure 10e, the Dv90 sizes for flow velocities of 0, 5.1 and 12.7 $m.s^{-1}$ are 30, 22 and 18 microns respectively. This occurs due to fewer collisions, as described above.
- Some impaction-pin nozzles exhibit bimodal distribution but only at *low* velocities. For example, in Figure 10 d, the red curve which represents a flow velocity of 0 $m.s^{-1}$, shows a bimodal pattern and only at larger Z locations. This is probably due to droplets recirculation. As velocity increases, the distribution becomes unimodal (see the blue line representing 5 $m .s^{-1}$ in Figure 10d.)
- The effect of gravity can be seen. For example in Figure 10a, 10b, and 10c, the Dv90 numbers are 22, 23 and 24 microns as the measurement location is moved from the top (R = -1) to the bottom (R = 1) of the plume. When we examine the behavior 12 cm from the nozzle (Figure 10d, 10e and 10f), the sizes are 32, 35 and 40 microns a larger change than was observed at R = 0.5 cm. As would be expected because more time has passed so the droplets have fallen further. Table 1 shows gravity effect on D32.
- In examining Figure 10f it is interesting to note that the volume frequency bell curve shifts to the left as the velocity increases. Again, this is due to the fact that as the velocity increases, the droplets follow the trajectory of the airflow and the probability of collision is lowered, resulting in smaller measured sizes. This effect can also be see but to a lesser extent in Figure 10c.
- Form these curves we see that from a droplet-size standpoint, increasing the airflow velocity above 1000 fpm (5.1 $m.s^{-1}$) does not reduce droplet sizes significantly. In most cases, gas turbine fogging nozzles are installed in a location in the inlet duct that has about this velocity. Locating the nozzles in a higher velocity region therefore would have little effect on droplet size.

EXPERIMENTAL STUDY OF IMPACTION-PIN AND SWIRL-JET NOZZLES

An extensive experimental study was conducted to examine the comparative performance of swirl-jet and impaction-pin fog nozzles in their application to gas turbine inlet air fogging. The study included both still-air tests and also wind tunnel experiments at different airflow velocities and for different nozzle-operating pressures. Some of the results are provided here along with comments.

Still-Air Experimental Analysis—Swirl-jet and Impaction-pin Nozzles

The experiment was done in still air, at an operating pressure of 2000 psig (137 barg) and a distance from the nozzle of 2 inches (5.08 cm). The two nozzles selected had similar orifice sizes (about 150 microns) but because of the greater losses in the swirl-jet nozzle, the flow rate for the impaction-pin nozzles was slightly higher (0.045 gpm, 0.170 liter/min for the impaction-pin and 0.036 gpm, 0.136 liter/min for the swirl-jet, both at 138 barg, 2000 psig).

The objective of these tests was to compare the general characteristics of two nozzle types[7]. Specific nozzle designs may differ slightly in performance details but the differences between a nozzle of a given design (i.e. swirl-or impaction-pin) with a given orifice diameter were not found to be significant. The droplet size (both SMD and Dv90) and the concentration volume (CV in ppm) were measured. The concentration volume quantifies the amount of water in the measurement volume. The experimental results are presented in Figure 11a (Swirl Nozzle) and in Figure 11b (Impaction Nozzle). The following observations may be made from Figure 11.

- For both types of nozzle, the droplet diameters are smaller in the center of the plume. As stated earlier, this occurs due to the airflow ingress effect as explained by Obakata and Long [11]. This clearly shows that results obtained from measurements taken only at the center of the spray plume are of little value.
- In examining the volume concentration (CV) one can see that for the swirl-jet (Figure 11a) the concentration is much less in the center due to the hollow cone effect. With the impaction-pin, there is more uniformity[8] as is seen in Figure 11b. This is due to the smaller droplet size produced. With a swirl-jet volume concentrations will become more homogeneous as the axial distance increases. (This effect can also be seen in Figure 8).

The impaction-pin makes smaller droplets than the swirl-jet; on average 6 micron smaller SMD and 7 micron smaller Dv90. This is primarily due to the fact that the straight-through orifice of the impaction-pin has less frictional loss than the swirl-jet nozzle, so jet velocity is higher and the resulting conical sheet is thinner, hence smaller droplets.

[7] These observations are based on numerous tests that were conducted on a wide range of makes and designs of swirl-jet and impaction-pin nozzles.

[8] There will be a small non-uniformity if measurements are made in line with the impaction-pin. However, this spatial effect is very small.

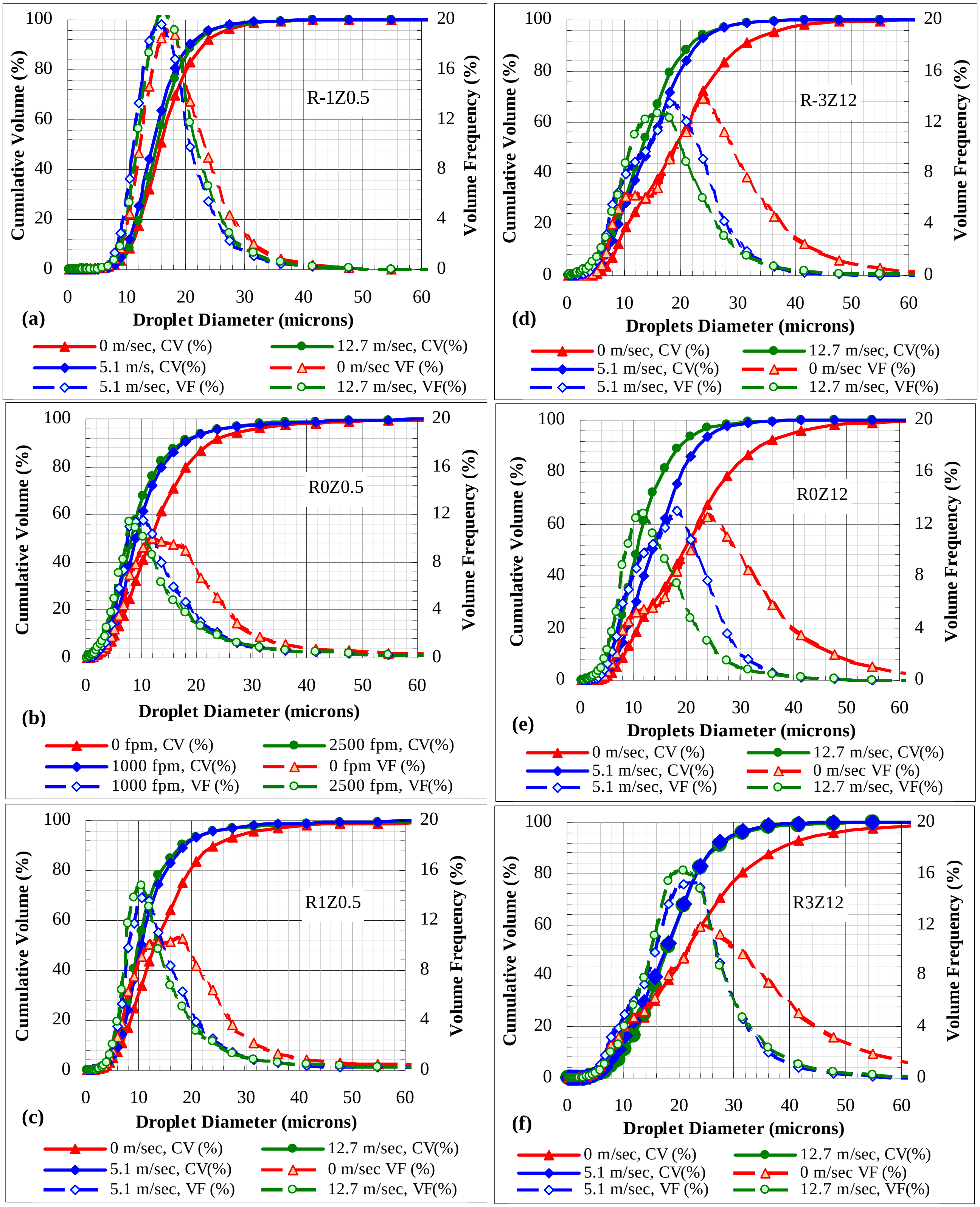

Figure 10. Experimental measurements showing influence of flow velocities and measurement distances on droplet sizes, Operating pressure: 138 barg

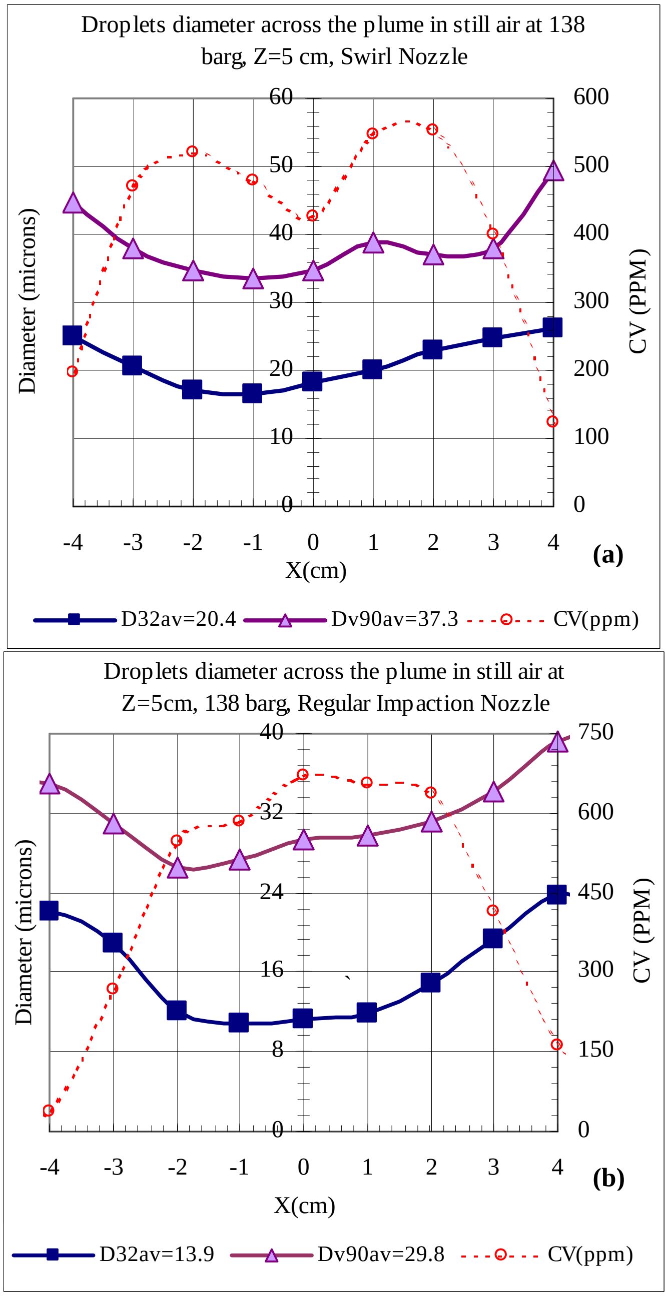

Figure 11 A and B. Experimental results of droplet size distribution in the plume for comparable swirl and impaction-pin nozzles

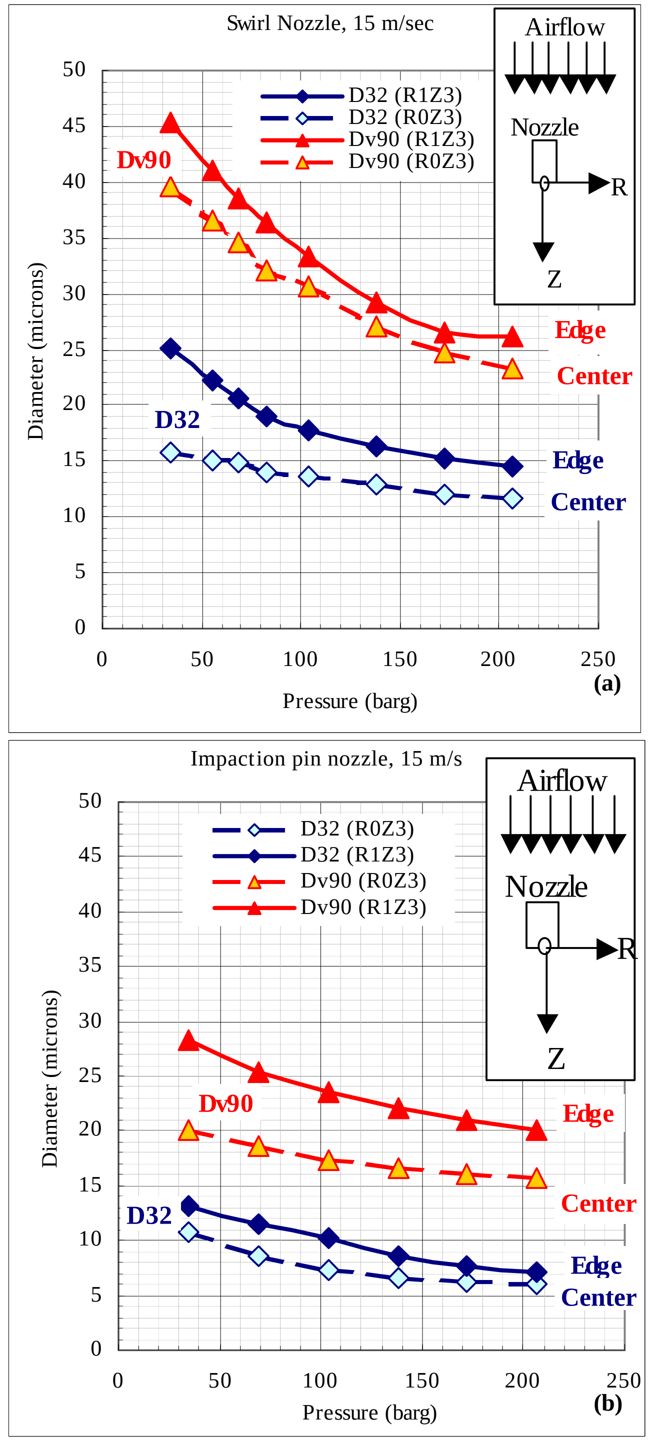

Figure 12. Experimental comparisons between impaction and swirl-jet nozzles at varying pressures. Measurements of both Dv90 and D32 have been taken at different locations in the plume. The graph shows that the impaction-pin nozzle provides smaller diameters regardless of where measured.

Wind Tunnel Experimental Analysis—Swirl-jet and Impaction-pin

To compare the performance of the two types of nozzles under airflow conditions, wind tunnel experiments were conducted. The airflow velocity was fixed at 15.2m/sec (3000 feet per minute). The nozzle operating pressure was varied between 21and 207 barg (300 and 3000 psig.). Measurements were taken in two positions in the plume. The first measurement was in the center of the plume 7.6 cm (3 inches) from the nozzle (R = 0 cm, Z = 7.6 cm). The second measurement location was also 7.6 cm from the nozzle orifice but at the edge of the plume (R = -2.5 cm, Z = 7.6 cm).

In examining Figure 12a and 12b for impaction-pin and swirl nozzles respectively, it is clear that impaction-pin droplet sizes make significantly smaller droplets even at higher airflow velocities. This is because the initial droplet sizes are smaller.

The following observations can be made from Figure 12:

- Droplet size decreases with an increase in pressure. While this hold true for both types of nozzle, the effect is more prevalent for swirl-jet nozzles until a pressure of about 1500 (103 barg) to 2000 psig (138 barg).

- For both nozzles, the curve of decreasing droplet size, with increasing pressure, begins to flatten out. This is due to the fact that there is an increase in droplet density and velocity near the nozzle orifice, which results in an increase in the probability of collision and coalescence. The effect is more pronounced with swirl-jet nozzles because the swirl cone begins to collapse with increasing pressure, resulting in both a thicker conical sheet (bigger droplets) and a higher droplet density and velocity (increased collision probability).
- For both nozzles, increasing the operating pressure above about 137 barg (2000 psig) does not result in significantly smaller droplets.

Even looking at the corresponding plumes visually (Figure 13) one can see that, for a swirl-jet nozzle, the cone shape can be distinctly seen, because the large droplets have higher penetration velocities. In examining Figure 13a, the smoke-like nature of the fog can be clearly seen. This provides a visual confirmation of the above data.

Figure 13, Plume characteristics of impaction-pin (left) and swirl-jet type nozzles (right) at operating pressure of 137 barg. The smoke-like nature of the impaction-pin nozzle is evident. The straight edge of the swirl-jet nozzle is indicative of the high momentum of the larger droplets, implying a much larger droplet size at the edges as also indicated by the graphs above

STANDARDIZATION OF MEASUREMENT METHODS AND DATA INTERPRETATION FOR FOGGING NOZZLES FOR GAS TURBINE APPLICATIONS

There is no extant standard of droplet size measurement for gas turbine inlet air fogging applications. ASTM has provided overall guidelines but no definitive approaches have been used in the fogging industry. The result is that the user is confronted by data that is impossible to reconcile. To overcome this problem, a new measurement standard has been developed and is proposed here. The measurement standard incorporates the following:

- Defines the instrumentation to be used.
- Defines locations where measurements are to be made.
- Defines the airflow velocities under which the measurements should be made.
- Defines the averaging approach and representative diameters to be used.
- Defines the final report format to be used.

Proposed Standard Of Droplet Size Measurement For Gas Turbine Fogging Applications.

We propose the following standard be adopted for the characterization of gas turbine inlet air fogging nozzles:

- Use the Malvern Spraytec system, or equivalent type instrument, for measurement of droplet size.
- Measurements should be made at flow velocity of 4.6 m/sec (15 ft/sec)[9].
- All tests should be conducted on a minimum of 5 randomly selected nozzles within error bars of 10%.
- Measure across the spray plume at a distance of 2 inches[10] (5 cm) from the orifice. Take as many readings as are required to cover the entire cross section of the plume.
- The measurement duration for each portion measured should be a least one-minute, to ensure measurement repeatability.
- Calculation of the diameter should be a weighted average performed by using the volume concentrations (ppm) derived at each measurement point. This calculation should be done on all representative diameters.
- Final report should include the Dv90 and D32 (Sauter Mean) diameters as these are the most significant for gas turbine operations.
- A single representative diameter (the Mee diameter) may be derived by averaging the Dv90 and D32 values.

The variation of the droplet size over a period of 5 minutes, for a given position in the spray, with a sampling frequency of 100 Hz is shown in Figure 14. The figure shows a relative variation of less than 5% in the droplet size, which indicates that the Malvern unit offers good repeatability. This sort of figure should be show for any droplet test to assure repeatability.

Further work is required to develop a comprehensive specification and testing procedure but the standard given is a good start. This standard was developed by and has been extensively used by Mee Industries for all its analytical work. It represents, in our opinion, the most detailed and accurate method for measuring and reporting droplet sizes for inlet fogging nozzles and will make it possible for turbine operators to make reasonable comparisons of different fog nozzles.

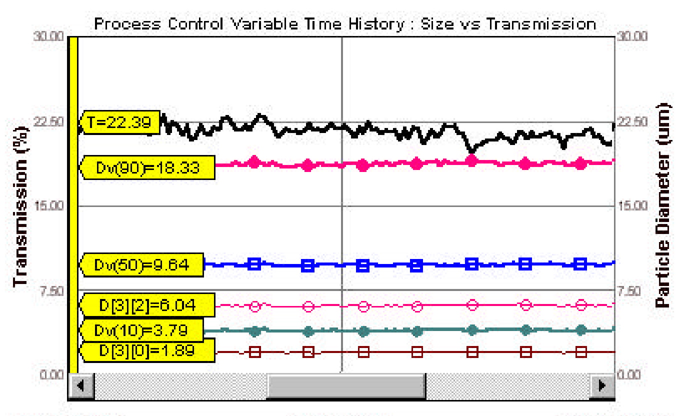

Figure 14. Data check for repeatability showing consistency in diameters

[9] This velocity has been chosen as a reasonable velocity for typical location of inlet fogging manifolds in gas turbine ducts.

[10] The distance of 2 inches (50.8mm) was been selected, as it is far enough away to allow for the measurement of any coalescence yet close enough to minimize the effect of evaporation.

CLOSURE

This paper has provided a comprehensive analysis of droplet sizing methodology, design aspects of fog nozzles and measurement approaches as they apply to gas turbine inlet air fogging. The importance of the correct use of representative droplet diameter definitions and proper measurement approaches have been covered, and the effect of airflow velocity on fog droplet behavior, and the resulting effect on final droplet sizes, has been described. Details of experimental studies comparing swirl-jet and impaction-pin nozzles have been provided. Extensive testing has shown that under all operating conditions, the droplet sizes of impaction-pin nozzles are smaller than swirl-jet nozzles with the same flow rate. Finally, a standard procedure for the characterization of fog droplets in the context of gas turbine inlet air fogging has been defined and proposed. Chaker et al [12, 13] provide further details on theoretical and practical considerations and fog behavior in gas turbine inlet air ducts.

ACKNOWLEDGMENTS

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APPENDIX A

Drop size equations:

Name	Formulae
Arithmetic mean diameter	$D_{10}=\frac{\sum_{i=1}^{p} n_i \times D_i}{\sum_{i=1}^{p} n_i}=\frac{n_1\times D_1+n_2\times D_2+.......+n_p\times D_p}{n_1+n_2+........+n_p}$
Surface mean diameter	$D_{20}=\sqrt{\frac{\sum_{i=1}^{p} n_i \times D_i^2}{\sum_{i=1}^{p} n_i}}=\sqrt{\frac{n_1\times D_1^2+n_2\times D_2^2+.......+n_p\times D_p^2}{n_1+n_2+........+n_p}}$
Volume mean diameter	$D_{30}=\sqrt[3]{\frac{\sum_{i=1}^{p} n_i \times D_i^3}{\sum_{i=1}^{p} n_i}}=\sqrt[3]{\frac{n_1\times D_1^3+n_2\times D_2^3+.......+n_p\times D_p^3}{n_1+n_2+........+n_p}}$
Absorption Diameter	$D_{21}=\frac{\sum_{i=1}^{p} n_i \times D_i^2}{\sum_{i=1}^{p} n_i\times D_i}=\frac{n_1\times D_1^2+n_2\times D_2^2+.......+n_p\times D_p^2}{n_1\times D_1+n_2\times D_2+........+n_p\times D_p}$
Evaporative Diameter	$D_{31}=\sqrt{\frac{\sum_{i=1}^{p} n_i \times D_i^3}{\sum_{i=1}^{p} n_i\times D_i^1}}=\sqrt{\frac{n_1\times D_1^3+n_2\times D_2^3+.......+n_p\times D_p^3}{n_1\times D_1^1+n_2\times D_2^1+........+n_p\times D_p^1}}$
Sauter mean diameter	$D_{32}=\frac{\sum_{i=1}^{p} n_i \times D_i^3}{\sum_{i=1}^{p} n_i\times D_i^2}=\frac{n_1\times D_1^3+n_2\times D_2^3+.......+n_p\times D_p^3}{n_1\times D_1^2+n_2\times D_2^2+........+n_p\times D_p^2}$